Class 10

Mathematics

Section-A (1 Mark each)

1. P (not E) = 1 – P(E) = 1 – 0.9 = 0.1

   Answer (a) 0.1

2. x + y = 1 and x – y = 5

   Adding both equation; x + y + x – y = 1 + 5 ⇒ 2x = 6 ⇒ x = 3

   Putting the value of x in equation (1), 3 + y = 1 ⇒ y = 1 – 3 ⇒ y = - 2

   Answer (a) (3, -2)

3. $\cos A = \frac{3}{5} = \frac{B}{H}$

   By Pythagoras' theorem,

   $H^2 = P^2 + B^2 \Rightarrow 5^2 = P^2 + 3^2 \Rightarrow P^2 = 25 - 9 = 16 \Rightarrow$ P = 4

   $\sin A = \frac{P}{H} = \frac{4}{5}$

   Answer (c) $\frac{4}{5}$

4. $\frac{AB}{DE} = \frac{BC}{FD}$

   $\Rightarrow \angle B = \angle D$

   Answer (d) $\angle B = \angle D$

5. -1, -1, -1, …..

   d = $a_2 - a_1$ = - 1 – (-1) = - 1 + 1 = 0

   Answer (a) 0

6. (p, q) and (-p, -q)

   Distance = $\sqrt{(-p-p)^2 + (-q-q)^2} = \sqrt{4p^2 + 4q^2} = 2\sqrt{p^2 + q^2}$

   Answer (c) $2\sqrt{p^2 + q^2}$

7. Odd number less than 5 = 1, 3

   P (Odd number less than 5) = $\frac{2}{6} = \frac{1}{3}$

   Answer is (a) $\frac{1}{3}$

8. 3x – y + 8 = 0 and 9x – ky + 24 = 0

   For coincident lines,

   $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{3}{9} = \frac{-1}{-k} = \frac{8}{24}$

$\Rightarrow \frac{1}{3} = \frac{1}{k} \Rightarrow k = 3$

Answer is (d) 3

9. 5, 8, 11, ….., 65

10^{th} term from the end

65, …., 11, 8, 5

a = 65, d = 5 – 8 = - 3

$a_{10} = a + (10 - 1)d = 65 + 9\times(-3) = 65 - 27 = 38$

Answer is (d) 38

10. Total persons = 15

Persons cannot swim = 6

Persons can swim = 15 – 6 = 9

P (Persons can swim) = $\frac{9}{15} = \frac{3}{5}$

Answer is (c) $\frac{3}{5}$

11. 2x + y = 3 and 4x + 2y = 6

$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$;

$\frac{b_1}{b_2} = \frac{1}{2}$;

$\frac{c_1}{c_2} = \frac{3}{6} = \frac{1}{2}$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Answer is (c) Infinitely many solutions

12. The distance of the point (-4, 5) from the y-axis is 4**.**

Answer is (a) 4

13. The probability of a sure event is 1.

Answer is (b) 1

14. $\frac{\tan^2 45^o - 1}{2\tan 45^o} = \frac{(1)^2 - 1}{2(1)} = \frac{0}{2} = 0$

Answer is (d) 0

15. ST∥QR,

By BPT, $\frac{PS}{SQ} = \frac{PT}{TR} \Rightarrow \frac{4.5}{1.5} = \frac{PT}{1.8} \Rightarrow PT = \frac{4.5}{1.5} \times 1.8 = 5.4$ cm

Answer is (a) 5.4 cm

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Answer is (a) $\frac{EF}{PR} = \frac{DE}{PQ}$

16. $96 = 2\times 2 \times 2 \times 2 \times 2 \times 3 = 2^5 \times 3$

The exponent of 2 = 5

Answer is (d) 5

17. centre is (-3, 2) and B (4, 1) and A (x, y)

$(-3, 2) = \left(\frac{4+x}{2}, \frac{1+y}{2}\right)$

$\frac{4+x}{2} = -3$ and $\frac{1+y}{2} = 2$

x = -6 -4 = - 10 and y = 4 – 1 = 3

Answer is (a) (-10, -3)

18. 3, 1, -1, …..

a = 3, d = 1 – 3 = - 2

$S_{12} = \frac{12}{2}[2 \times 3 + (12-1)(-2)] = 6[6-22] = 6\times(-16) = -96$

Answer is (c) – 96

Directions for questions 19 & 20:

19. (a)

20. (c)

Section-B (2 Marks each)

21. $2\tan^2 45^o + \sin^2 60^o + \cos^2 45^o$

$= 2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2$

$= 2 + \frac{3}{4} + \frac{1}{2}$

$= \frac{8+3+2}{4} = \frac{13}{4}$

22. $65 = 5 \times 13$

$26 = 2 \times 13$

HCF = 13

Or

$\sqrt{3}$ and $\sqrt{7}$

$\sqrt{3}$ = 1.72 and $\sqrt{7}$ = 2.64

Rational Number = $\sqrt{4}$ = 2

Irrational Number = $\sqrt{5}$

23. Let ratio be k : 1.

Let A (2, 0) and B (0, 4)

point P (1, 2).

Coordinate of P = $\left(\frac{m\times x_2+n\times x_1}{m+n}, \frac{m\times y_2+n\times y_1}{m+n}\right)$

(1, 2) = $\left(\frac{k\times 0+1\times 2}{k+1}, \frac{k\times 4+1\times 0}{k+1}\right)$

1 = $\frac{k\times 0+1\times 2}{k+1}$

k + 1 = 0 + 2

k = 1

Ratio is 1 : 1

Or

(-1, 0) and (4, 1)

ratio 2:1

Coordinate of Point = $\left(\frac{m\times x_2+n\times x_1}{m+n}, \frac{m\times y_2+n\times y_1}{m+n}\right)$

= $\left(\frac{2\times 4+1\times(-1)}{2+1}, \frac{2\times 1+1\times 0}{2+1}\right)$

= $\left(\frac{7}{3}, \frac{2}{3}\right)$

24. Total bulbs = 30

Defective bulbs = 8

good bulbs = 30 – 8 = 22

P(good bulbs) = $\frac{22}{30}$ = $\frac{11}{15}$

25. x-axis ⟹ y = 0

Let Point P (x, 0) and A (0, 4)

PA =2$\sqrt{5}$ units

$\sqrt{(x-0)^2+(4-0)^2}$ = 2$\sqrt{5}$

squaring on both sides,

$x^2+16=20$

x^2= 4

x = ± 2

So, Point P (2, 0) or (-2,0)

Section-C (3 Marks each)

26. LHS

= $\sqrt{\frac{\sin A+1}{1-\sin A}}$

= $\sqrt{\frac{\sin A+1}{1-\sin A} \times \frac{1+\sin A}{1+\sin A}}$

= $\sqrt{\frac{(1+\sin A)^2}{1-\sin^2 x}}$

= $\sqrt{\frac{(1+\sin A)^2}{\cos^2 A}}$

= $\frac{1+\sin A}{\cos A}$ = $\frac{1}{\cos A} + \frac{\sin A}{\cos A}$

$= \sec A + \tan A$ = RHS

Or

LHS

= $\frac{\tan A - \sin A}{\tan A + \sin A}$

= $\frac{\frac{\sin A}{\cos A} - \sin A}{\frac{\sin A}{\cos A} + \sin A}$

= $\frac{\sin A\left(\frac{1}{\cos A} - 1\right)}{\sin A\left(\frac{1}{\cos A} + 1\right)}$

= $\frac{\frac{1}{\cos A} - 1}{\frac{1}{\cos A} + 1}$

$= \frac{\sec A - 1}{\sec A + 1}$ = RHS

27. Assume that $\sqrt{2}$ be an rational number.

$\sqrt{2}$ = $\frac{a}{b}$, where a and b are coprime.

Suppose a and b have a common factor other than 1.

$\sqrt{2}$ = $\frac{a}{b}$

b$\sqrt{2}$ = a

Squaring on both sides,

$2b^2$= a^2

Therefore, 2 divides a 2 .

Also, 2 divides a.

Putting a = 2c for some integer c.

$2b^2 = 4c^2 \Rightarrow b^2 = 2c^2$

This means that 2 divides b^2, and so 2 divides b

Therefore, a and b have at least 2 as a common factor.

But this contradicts the fact that a and b have no common factors other than 1.

So our assumption is incorrect that $\sqrt{2}$ is rational.

Hence, we conclude that $\sqrt{2}$ is irrational.

28. x + 3y = 6 … (1)

and 5x – 6y = 30 … (2)

From equation (1),

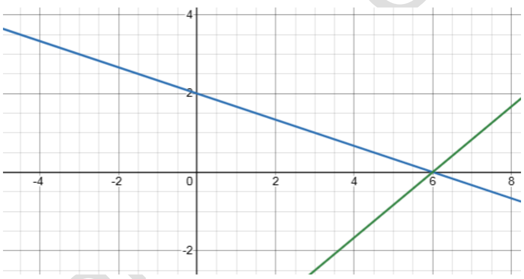
x + 3y = 6

X	0	6
Y	2	0

From equation (2),

5x – 6y = 30

X	0	6
Y	-5	0



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(k + 1)x + ky = 6 and 4y + 6x = 12

$a_1 = (k + 1)$, $b_1 = k$, $c_1 = 6$

$a_2 = 6$, $b_2 = 4$, $c_2 = 12$

For infinitely many solutions.

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\frac{k+1}{6} = \frac{k}{4} = \frac{6}{12}$

$\frac{k+1}{6} = \frac{6}{12}$ or $\frac{k}{4} = \frac{6}{12}$

$\Rightarrow \frac{k}{4} = \frac{1}{2}$

$\Rightarrow$ k = 2

29. First 50 positive integers divisible by 4

4, 8, 12, …. 48

a = 4, d = 8 – 4 = 4

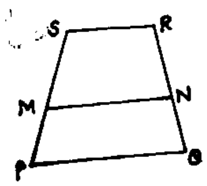
$S_{50} = \frac{50}{2}[2 \times 4 + (50-1)4]$

$= 25\ [8 + 196]$

= 25(204)

= 5100

30.



$PQ \| SR$ and $MN \| PQ$

So, $PQ \| MN$ and $MN \| SR$,

If three (or more) parallel lines are cut by two transversals, then the corresponding segments on the transversals are proportional.

$\Rightarrow \frac{SM}{MP} = \frac{RN}{NQ}$

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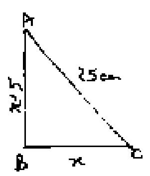

Let BC = x cm and AB = (x + 5) cm

AC = 25 cm

By Pythagoras' theorem,

$AC^2 = AB^2 + BC^2$

$\Rightarrow 25^2 = (x+5)^2 + x^2$

$\Rightarrow 625 = x^2 + 25 + 10x + x^2$

$\Rightarrow 2x^2 + 10x + 25 - 625 = 0$

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⇒ $2x^2$ + 10x – 600 = 0

⇒ $2(x^2 + 5x - 300)$ = 0

⇒ $x^2 + 5x - 300$ = 0

⇒ $x^2 + 20x - 15x - 300$ = 0

⇒ (x + 20)(x - 15) = 0

⇒ x = 15 [x ≠-20 it is impossible]

So BC = 15 cm and AB = 15 + 5 = 20 cm

31. $\sec\theta = \frac{13}{5} = \frac{H}{B}$

By Pythagoras theorem,

$H^2 = P^2 + B^2 \Rightarrow 13^2 = P^2 + 5^2 \Rightarrow P^2 = 169 - 25 = 144$

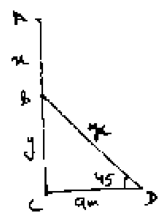
P = 12

$$\sin\theta = \frac{P}{H} = \frac{12}{13}; \cos\theta = \frac{B}{H} = \frac{5}{13}; \tan\theta = \frac{P}{B} = \frac{12}{5}$$

$$\csc\theta = \frac{H}{P} = \frac{13}{12}, \cot\theta = \frac{B}{P} = \frac{5}{12}$$

Section-D (5 Marks each)

32.



Let AB = x m and BC = y m

CD = 9 m

In ΔBCD,

$\frac{BC}{CD} = \tan 45^o$

⇒ $\frac{y}{9}$ = 1 ⇒ y = 9 m

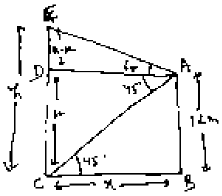
Again, In ΔBCD,

$\frac{CD}{BD} = cos\, 45^o$

⇒ $\frac{9}{x} = \frac{1}{\sqrt{2}}$ ⇒ x = $9\sqrt{2}$ m

Height of the tree = AC = AB + BC = x + y = $9\sqrt{2} + 9 = 9(\sqrt{2} + 1)$m

Or



In ΔABC,

$\frac{AB}{BC}$ = tan 45^o

⇒ $\frac{12}{x}$ = 1 ⇒ x = 12 m

and,

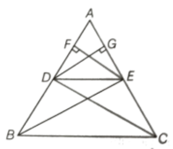
In ΔADE,

$\frac{ED}{DA}$ = tan 60^o

⇒ $\frac{h-12}{x}$ = $\frac{\sqrt{3}}{1}$ ⇒ h – 12 = x$\sqrt{3}$ ⇒ h = 12$\sqrt{3}$ + 12 = 12 ($\sqrt{3}$ + 1) m

So, height of the tower = 12 ($\sqrt{3}$ + 1) m

33. Given: ABC in which a line DE parallel to BD intersect AB to D and AC at E.



To prove: $\frac{AD}{DB}$ = $\frac{AE}{EC}$

Construction : Join BE, CD and draw EF⊥AB and DG⊥AC.

Proof Here $\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{\frac{1}{2} \times AD \times EF}{\frac{1}{2} \times DB \times EF}$ [∵ area of triangle= $\frac{1}{2}$ × base × height]

$= \left(A\frac{D}{D}B\right)$...(i)

similarly, $\frac{ar(\Delta ADE)}{ar(\Delta DEC)} = \frac{\frac{1}{2} \times AE \times GD}{\frac{1}{2} \times EC \times GD} = \frac{AE}{EC}$....(ii)

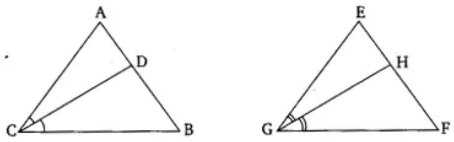
Now since, ΔBDE and ΔDEC lie between the same parallel DE and BC and on the same base DE.

So, $ar(\Delta BDE) = ar(\Delta DEC)$...$(iii)$

From Eqs. (i), (ii) and (iii), $\frac{AD}{DB} = \frac{AE}{EC}$ Hence proved

Or

ΔABC and ΔEFG are given below.



(i) ΔABC ~ ΔEFG [Given]

and ∠ACB = ∠EGF [Given]

∴ ∠BCD = ∠EGH ...(i)

[Bisectors of equal angles are equal]

Also ∠B = ∠E

∠DCB = ∠HGE ...(ii)

From equations (i) and (ii), we get:

ΔDCB ~ ΔHGE [By AA similarity]

(ii) similarity, ΔDCA ~ ΔHGF

34. Let the fraction be $\frac{x}{y}$.

ATQ,

$\frac{x+2}{y} = 1$ ⇒ x + 2 = y ⇒ x – y = - 2 …. (1)

and $\frac{x+3}{y+3} = \frac{5}{6}$ ⇒ 6x + 18 = 5y + 15 ⇒ 6x – 5y = - 3 ….(2)

From equation (1), x – y = -2 ⇒ x = - 2 + y ….(3)

Putting the value of x in equation (2),

6(- 2 + y) – 5y = - 3 ⇒ - 12 + 6y – 5y = - 3

⇒ y = - 3 + 12 ⇒ y = 9

Putting the value of y in equation (3), x = - 2 + 9 = 7

So, fraction = $\frac{7}{9}$

35. Let A(-2, -1), B(0, 1), C(2, -1) and D(0, -3)

AB = $\sqrt{(0+2)^2 + (1+1)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

BC = $\sqrt{(2-0)^2 + (-1-1)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

CD = $\sqrt{(0-2)^2 + (-3+1)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

DA = $\sqrt{(0+2)^2 + (-3+1)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

And,

Diagonal AC = $\sqrt{(2+2)^2 + (-1+1)^2} = \sqrt{16} = 4$

Diagonal BD = $\sqrt{(0+0)^2 + (-3-1)^2} = \sqrt{16} = 4$

Here, AB = BC = CD = DA = $2\sqrt{2}$

Diagonal AC = BD = 4

Therefore, the given points are the vertices of a square.

Section-E

36.

Let the trees be numbered 1 to 20 from the nearest to farthest.

Distance of the nth tree from the tank (one-way) is

$a_n = 25 + (n - 1)10$

The member walks from the tank to the tree and returns, so the round-trip distance for the k^{th} tree is

$R_n = 2a_n = 2\,(25 + (n - 1)10)$

(i) The distance travelled by the member to water nearest tree and back to the tank

$n = 1$, $R_1 = 2\times25 = 50$ m

(ii) The distance travelled by the member to water second tree and back to the tank.

$n = 2$, $R_2 = 2(25 + 10) = 70$ m

(iii) The distance travelled by the member to water tenth tree and back to the tank.

$n = 10$, $R_{10} = 2(25 + 90) = 230$ m

Or

$a = 25$m and $d = 10$ m

$a_n = 2\,(25 + (n - 1)10) = 50 + 20(n - 1) = 30 + 20n$

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(i) 50, 70, 90, 110, 130, 150, 170…..

(ii) $a = 50$, $d = 20$, $a_n = 250$

$a_n = 250 \Rightarrow a + (n - 1)d = 250$

$\Rightarrow 50 + (n - 1)20 = 250 \Rightarrow (n-1)20 = 200 \Rightarrow n = 11$

(iii) The total amount she saves in first 10 weeks

$S_{10} = \frac{10}{2}[2a + (10 - 1)d] = 5\,[2(50) + 9(20)] = 5(280) = 1400$

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Or

$S_{15} = \frac{15}{2}[2a + (15 - 1)d] = \frac{15}{2}[2(50) + 14(20)] = \frac{15}{2}(380) = 2850$

37. Total cards = 20

(i) an odd number = 3,5,7,9,11,13,15,17,19,21

P (an odd number) = $\frac{10}{20}$ = 0.5

(ii) a number smaller than 10 = 3,4,5,6,7,8,9

P (a number smaller than 10) = $\frac{7}{20}$ = 0.35

(iii) a perfect square number = 4, 9, 16

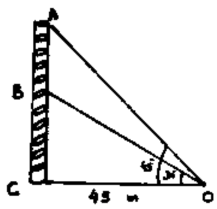
P (a perfect square number) = $\frac{3}{20}$ = 0.15

Or

a number divisible by 3 = 3,6,9,12,15,18,21

P (a number divisible by 3) = $\frac{7}{20}$ = 0.35

38.



In ΔACO,

$\frac{AC}{CO} = \tan\theta$

$\Rightarrow \frac{AC}{45} = \tan 45^o$

$\Rightarrow \frac{AC}{45} = 1$

$\Rightarrow$ AC = 45 m

In ΔBCO,

$\frac{OC}{OB} = \cos\theta$

$\Rightarrow \frac{45}{OB} = \cos 30^o$

$\Rightarrow \frac{45}{OB} = \frac{\sqrt{3}}{2}$

$\Rightarrow OB = \frac{90}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{90\sqrt{3}}{3} = 30\sqrt{3}$m

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Again,

In ΔBCO,

$\frac{BC}{CO} = tan\,\theta$

$\Rightarrow \frac{BC}{45} = tan\,30^o$

$\Rightarrow \frac{BC}{45} = \frac{1}{\sqrt{3}}$

$\Rightarrow$ BC = $\frac{45}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{45\sqrt{3}}{3} = 15\sqrt{3}$m

Now, AB = AC - BC = 45 - $15\sqrt{3} = 15(3 - \sqrt{3})$m

And,

ΔOCA = $\frac{1}{2}$ ×AC×CO = $\frac{1}{2} \times 45 \times 45 = \frac{2025}{2}$m

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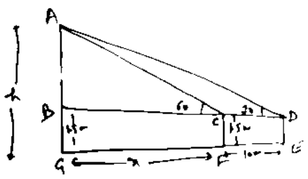

Let AG = h m, and GF = BC = x m and CD = FE = 10 m

AB = (h – 1.5) m

In ΔABC,

$\frac{AB}{BC}$ = $\tan 60^o$

$\frac{h-1.5}{x} = \sqrt{3} \Rightarrow$ h – 1.5 = $x\sqrt{3} \Rightarrow$ h = 1.5 + $x\sqrt{3}$.... (1)

In ΔABD,

$\frac{AB}{BD}$ = $\tan 30^o$

$\frac{h-1.5}{x+10} = \frac{1}{\sqrt{3}} \Rightarrow$ h – 1.5 = $\frac{(x+10)}{\sqrt{3}} \Rightarrow$ h = 1.5 + $\frac{(x+10)}{\sqrt{3}}$.... (2)

From (1) and (2),

1.5 + $x\sqrt{3}$ = 1.5 + $\frac{(x+10)}{\sqrt{3}}$

$\Rightarrow x\sqrt{3} = \frac{(x+10)}{\sqrt{3}} \Rightarrow$ 3x = x +10 $\Rightarrow$ 2x = 10 $\Rightarrow$ x = 5 m

Putting the value of x in eq (1), h = 1.5 + $5\sqrt{3}$

(i) the angle of elevation of top of the pole when Raju was closer = 60^o

(ii) 10 m

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(iii) Height of the pole = $1.5 + 5\sqrt{3} = 1.5 + 5(1.73) = 10.15$ m

or

The distance of the initial position of Raju from the base of the pole = $x = 5$ m

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